The irradiation of a medium by short intense x-ray (XUV) FEL pulses can result in population inversion. The ensuing spontaneous emission from one atom in such a medium can stimulate emission from other atoms. This self-amplified process can lead to short and intense emission of radiation that is referred to as superfluorescence. In order to correctly describe corresponding phenomena, one needs a quantum mechanical treatment of both the electromagnetic field and the atomic systems. However, due to the large number of interacting atoms and a continuum of field modes, an exact solution is not possible. In order to obtain tractable models, certain approximations thus have to be introduced, which are specific for each parameter range. Hence, the methods developed in the optical domain cannot be generalized straightforwardly. In particular, for x-ray (XUV) cases, the pumping and/or decay due to mechanisms other than spontaneous emission can take place at timescales comparable to the superfluorescent emission. Consequently, one needs to treat all these processes on the same footing. Here - within the scope of certain approximations (two-level model for resonant atomic transitions, 1D model for field propagation, factorization of triple operator products) - we formulate a system of integro-differential equations for field and matter correlation functions. This system of equations allows us to describe the crossover between spontaneous emission, amplified spontaneous emission and superfluorescence.

As an application, we have used our approach to model the superfluorescent emission from a Xe gas sample irradiated by FLASH. For the strongest spectral line, an amplification over 4 orders of magnitude - reaching saturated emission - was observed experimentally and successfully interpreted based on the developed formalism. Moreover, an increase of the emission line-width as a function of emission yield was experimentally demonstrated that – taking into account for the instrumental resolution – corresponds to results of our theoretical modeling.